# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2298

A MODIFICATION TO THIN-AIRFOIL-SECTION THEORY, APPLICABLE TO ARBITRARY AIRFOIL SECTIONS, TO ACCOUNT FOR THE EFFECTS OF THICKNESS ON THE LIFT DISTRIBUTION

By David Graham

Ames Aeronautical Laboratory
Moffett Field, Calif.

Washington
February 1951

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2298

# A MODIFICATION TO THIN-AIRFOIL-SECTION THEORY, APPLICABLE TO ARBITRARY AIRFOIL SECTIONS, TO ACCOUNT FOR THE EFFECTS OF THICKNESS ON THE LIFT DISTRIBUTION

By David Graham

## SUMMARY

The method of accounting for the effect of thickness on the lift distribution given in NACA Report 833, 1945, has been modified. Comparison of lift distributions calculated by the methods of the present report and NACA Report 833 together with experimental distributions are presented. The results show that the modification improves the predictions of lift distributions for both angle of attack and camber.

## INTRODUCTION

There are a number of methods available for the prediction of the lift distribution on given airfoil sections in incompressible flow (e.g., references 1 through 6) Of the methods cited, reference 3 alone presents a method which is relatively rapid and is applicable to arbitrary airfoils. The accuracy of the method of reference 3, however, decreases with increasing amounts of section camber and/or thickness, particularly in the region of the airfoil nose. The purpose of the present report is to describe a modification to that part of the theory of reference 3 which deals with the lift distribution associated with angle of attack and/or camber. The modification is directed at improving the accuracy of the method of reference 3 without decreasing its simplicity. No change is made in that part of the theory of reference 3 dealing with the base profile. The discussion that follows presupposes a familiarity with the contents and method of reference 3.

## METHOD

The following relations for an airfoil of infinitesimal thickness are presented in reference 3 (the notation used herein, unless otherwise noted, is the same as given in reference 3 and is presented in the appendix):

1. The velocity $v$ induced at any point $x_o$ on the camber line, due to all the vortices distributed along the camber line, is

$$v_{(x_o)} = \frac{1}{2\pi} \int_o^c \frac{\frac{d_o\Gamma}{dx}\,dx}{x-x_o} \qquad \text{(equation (1) of reference 3)} \qquad (1)$$

2. The boundary condition that the direction of the flow close to the camber line must be parallel to the surface of the camber line yields the relation

$$\frac{v}{V_o} = \frac{dy_c}{dx} - \alpha \qquad \text{(equation (2) of reference 3)} \qquad (2)$$

3. The distribution of vorticity is assumed to be

$$\frac{d_o\Gamma}{dx} = 2\,V_o\left(A_o'\cot\frac{\theta}{2} + \sum_1^\infty A_n \sin n\theta\right)$$

(from equation (4) of reference 3) (3)

4. The lift at a point along the chord is

$${}_oP = \frac{\rho V_o \frac{d_o\Gamma}{dx}}{q} \qquad \text{(equation (14) of reference 3)} \qquad (4)$$

5. The total lift is

$${}_oL = \int_o^c \rho V_o \frac{d_o\Gamma}{dx}\,dx \qquad (5)$$

The coefficient $A_o'$ in equation (3) is shown to vary only with angle of attack, while the coefficients $A_n$ are independent of the angle of attack and are functions of the mean camber-line shape only. Thus the part of the lift distribution which varies with angle of attack, called the additional-lift distribution, and the part of the lift distribution which is a function of the mean camber-line shape only, called the basic-lift distribution, can be treated separately.

As stated in reference 3, the preceding equations were derived from a theory for thin airfoils where the local velocity at each elemental vortex along the camber line was taken to be free-stream velocity $V_o$. Reference 3 suggests that, for the airfoil of finite thickness, a more accurate prediction of the lift distribution would be obtained if the local velocity at each vortex station were taken to be the local velocity $V_f$ at the surface of the base profile at the same chordwise

station. Reference 3 approximates the effect of the local velocity by adjusting the local lift at each point by the ratio of local to free-stream velocity. That is,

$$P = {}_oP\left(\frac{V_f}{V_o}\right) \qquad \text{(equation (28) of reference 3)} \qquad (6)$$

Equations (4) and (5) then become, for the airfoil of finite thickness,

$$P = \frac{\rho V_f \frac{d\,{}_o\Gamma}{dx}}{q} \qquad (7)$$

and

$$L = \int_0^c \rho V_f \frac{d\,{}_o\Gamma}{dx}\,dx \qquad (8)$$

The approximation given by equation (6) led to improved accuracy of the lift-distribution calculation. No attempt was made in reference 3 to improve further the accuracy by solving for a new vorticity distribution using the new boundary condition which would result if $V_f$ were substituted for $V_o$ in equation (2).

The need for further improvement in the prediction of pressure distributions, with particular emphasis on accounting for the effects of thickness - or in effect to account for the fact that $V_f$ becomes markedly different from $V_o$ - has made a re-examination of the problem necessary. In an attempt to calculate more accurately the effects of thickness on the chordwise load distribution, the boundary condition given by equation (2) is altered so that it becomes

$$\frac{v}{V_f} = \frac{dy_c}{dx} - \alpha \qquad (9)$$

With this new boundary condition a solution for the vorticity distributions can now be made in a manner similar to that used in reference 3.

The expression for the vorticity distribution is again assumed to be given by equation (3), but now equation (9) is used to define the coefficients of the series. From equations (1), (3), and (9) it can be shown that

$$\frac{V_f}{V_o}\left(\frac{dy_c}{dx} - \alpha\right) = -A_o' + \sum_1^\infty A_n \cos n\theta \qquad (10)$$

The coefficients are given by

$$A_o' = -\frac{1}{\pi}\int_0^\pi \frac{V_f}{V_o}\left(\frac{dy_c}{dx} - \alpha\right) d\theta \qquad (11)$$

and

$$A_n = \frac{2}{\pi} \int_0^{\pi} \frac{V_f}{V_o} \left( \frac{dy_c}{dx} - \alpha \right) \cos n\theta \, d\theta \tag{12}$$

The coefficients can each be separated into two parts, one part a function only of the camber-line shape, and the other part a function only of the angle of attack. Following previous work, the vorticity distribution, ascribed to the camber-line shape, is that existing when there is zero vorticity at the nose of the airfoil and is called the basic distribution. The angle of attack at which this distribution exists is called the ideal angle $\alpha_i$. Therefore, the total angle $\alpha$ is written as the sum of the ideal and additional angles of attack, as follows

$$\alpha = \alpha_i + \alpha_a$$

Equations (11) and (12) can be written as

$$A_o' = -\frac{1}{\pi} \int_0^{\pi} \frac{V_f}{V_o} \left( \frac{dy_c}{dx} - \alpha_i \right) d\theta + \frac{1}{\pi} \int_0^{\pi} \frac{V_f}{V_o} \alpha_a \, d\theta \tag{13}$$

and

$$A_n = \frac{2}{\pi} \int_0^{\pi} \frac{V_f}{V_o} \left( \frac{dy_c}{dx} - \alpha_i \right) \cos n\theta \, d\theta - \frac{2}{\pi} \int_0^{\pi} \frac{V_f}{V_o} \alpha_a \cos n\theta \, d\theta \tag{14}$$

It is now possible to write the expressions for the coefficients corresponding to basic and additional vorticity. For basic vorticity these are

$$A_{o_b}' = -\frac{1}{\pi} \int_0^{\pi} \frac{V_f}{V_o} \left( \frac{dy_c}{dx} - \alpha_i \right) d\theta \tag{15}$$

and

$$A_{n_b} = \frac{2}{\pi} \int_0^{\pi} \frac{V_f}{V_o} \left( \frac{dy_c}{dx} - \alpha_i \right) \cos n\theta \, d\theta \tag{16}$$

The coefficients for the additional vorticity are

$$A_{o_a}' = \frac{1}{\pi} \int_0^{\pi} \frac{V_f}{V_o} \alpha_a \, d\theta \tag{17}$$

and

$$A_{n_a} = -\frac{2}{\pi} \int_0^{\pi} \frac{V_f}{V_o} \alpha_a \cos n\theta \, d\theta \tag{18}$$

Thus the basic- and additional-vorticity distributions for the airfoil of finite thickness can be expressed as

$$\left(\frac{d\Gamma}{dx}\right)_b = 2V_o\left(A_{o_b}' \cot\frac{\theta}{2} + \sum_1^{\infty} A_{n_b} \sin n\theta\right) \tag{19}$$

and

$$\left(\frac{d\Gamma}{dx}\right)_a = 2V_o\left(A_{o_a}' \cot\frac{\theta}{2} + \sum_1^{\infty} A_{n_a} \sin n\theta\right) \tag{20}$$

## Basic-Lift Distribution

Since the basic vorticity is zero at the nose of the airfoil it is required that the coefficient $A_{o_b}'$ be equal to zero. This requirement fixes the value of $\alpha_i$ which is necessary for the evaluation of the coefficients $A_{n_b}$. Setting equation (15) for $A_{o_b}'$ equal to zero, that is,

$$A_{o_b}' = -\frac{1}{\pi}\int_0^{\pi} \frac{V_f}{V_o}\left(\frac{dy_c}{dx} - \alpha_i\right) d\theta = 0$$

the value of $\alpha_i$ is obtained as

$$\alpha_i = \frac{\int_0^{\pi} \frac{V_f}{V_o}\frac{dy_c}{dx}\, d\theta}{\int_0^{\pi} \frac{V_f}{V_o}\, d\theta} \tag{21}$$

The basic-vorticity distribution is now written as

$$\left(\frac{d\Gamma}{dx}\right)_b = 2V_o \sum_1^{\infty} A_{n_b} \sin n\theta \tag{22}$$

The basic lift is derived from equations (7), (16), and (22) as

$$P_{b_o} = 4\left(\frac{V_f}{V_o}\right)_o \frac{2}{\pi}\int_0^{\pi} \sum_1^{\infty} \frac{V_f}{V_o}\left(\frac{dy_c}{dx} - \alpha_i\right) \cos n\theta \sin n\theta_o\, d\theta \tag{23}$$

which can also be written

$$P_{b_o} = 4\left(\frac{V_f}{V_o}\right)_o \left[-\frac{1}{2\pi}\int_0^{2\pi} \frac{V_f}{V_o}\frac{dy_{c_b}}{dx} \cot\left(\frac{\theta-\theta_o}{2}\right) d\theta\right] \tag{24}$$

where

$$\left(\frac{V_f}{V_o}\frac{dy_{c_b}}{dx}\right)_{\pi+\theta} = \left(\frac{V_f}{V_o}\frac{dy_{c_b}}{dx}\right)_{\pi-\theta}$$

and

$$\frac{dy_{c_b}}{dx} = \frac{dy_c}{dx} - \alpha_i$$

In a similar manner, the camber required for a specified basic lift is obtained as

$$\left(\frac{dy_{c_b}}{dx}\right)_o = \left(\frac{V_o}{V_f}\right)_o \left[\frac{1}{2\pi}\int_0^{2\pi} \frac{V_o}{V_f}\frac{P_b}{4}\cot\left(\frac{\theta-\theta_o}{2}\right) d\theta\right] \tag{25}$$

where

$$\left(\frac{V_o}{V_f}\frac{P_b}{4}\right)_{\pi+\theta} = -\left(\frac{V_o}{V_f}\frac{P_b}{4}\right)_{\pi-\theta}$$

The accuracy of the basic-lift distribution calculated by use of equation (24) will be shown to be an improvement over the distribution calculated by the expression given in reference 3.

## Additional-Lift Distribution

The additional lift can be derived from equations (7), (17), (18), and (20). At the point $\theta_o$ the additional lift is

$$P_{a_o} = 4\left(\frac{V_f}{V_o}\right)_o \alpha_a \left[\left(\cot\frac{\theta_o}{2}\right)\frac{1}{\pi}\int_0^{\pi} \frac{V_f}{V_o} d\theta - \frac{2}{\pi}\int_0^{\pi} \sum_1^{\infty} \frac{V_f}{V_o}\cos n\theta \sin n\theta_o \, d\theta\right] \tag{26}$$

Use of equation (26) to express the additional-lift distribution results in a loss in computational simplicity as compared with the expression for the additional lift as given in reference 3. In addition, a comparison with experimental results indicates a loss in accuracy. However, a simple and adequate method for calculating the additional-lift distribution has been found by using a different approach. Throughout the remainder of the report it will be assumed that the basic lift is evaluated by the method just presented, and given by equation (24), and the additional-lift distribution is evaluated by the following method.

The additional-vorticity distribution of the infinitesimally thin airfoil, given by

$$\left(\frac{d\ {}_o\Gamma}{dx}\right)_a = 2V_o\ A_o'\ \cot\frac{\theta}{2} \tag{27}$$

is adjusted in proportion to the change in local velocity from $V_o$ to $V_f$ at each vortex station. The resulting expression for the additional-vorticity distribution of the finite-thickness airfoil is

$$\left(\frac{d\Gamma}{dx}\right)_a = \frac{V_f}{V_o}\left(\frac{d\ {}_o\Gamma}{dx}\right)_a = 2V_f\ A_o'\ \cot\frac{\theta}{2} \tag{28}$$

The additional-lift distribution is obtained by direct substitution of this expression for the vorticity distribution in the expressions for the local additional lift and the total additional-lift coefficient. The latter two expressions are, for the finite-thickness airfoil, respectively,

$$P_{a_o} = \frac{\rho\ V_{f_o}\left(\frac{d\Gamma}{dx}\right)_{a_o}}{q} \tag{29}$$

and

$$c_{l_a} = \int_o^c \frac{\rho\ V_f\left(\frac{d\Gamma}{dx}\right)_a dx}{q\ c} \tag{30}$$

The expression that is obtained for the additional lift per unit total additional-lift coefficient is

$$\frac{P_{a_o}}{c_{l_a}} = \frac{2\left(\frac{V_f}{V_o}\right)_o^2 \cot\frac{\theta_o}{2}}{\int_o^\pi\left(\frac{V_f}{V_o}\right)^2 (1 + \cos\theta)\ d\theta} \tag{31}$$

It may be noted that equation (31) can also be written in a form similar to equation (56) of reference 3, that is,

$$\left(\frac{P_{a_o}}{c_{l_a}}\right) = \frac{P_{a_o}\left({}_oc_{l_a} = 1\right)}{c_{l_a}\left({}_oc_{l_a} = 1\right)} \tag{32}$$

where now

$$P_{a_o}\left({}_oc_{l_a} = 1\right) = \left(\frac{V_f}{V_o}\right)_o^2 \left(\frac{{}_oP_a}{{}_oc_{l_a}}\right)_o$$

rather than

$$P_{a_o}\left({}_oc_{l_a} = 1\right) = \left(\frac{V_f}{V_o}\right)_o \left(\frac{{}_oP_a}{{}_oc_{l_a}}\right)_o \quad \text{(equation (55) of reference 3)}$$

The accuracy of additional-lift distributions calculated by use of equation (31) will be indicated later by comparing them with experimental lift distributions.

## METHOD OF APPLICATION

In applying the method of this report for the calculation of the lift distribution on an airfoil section, it is necessary to know the velocity distribution over the base profile. There are several ways to obtain the required base-profile velocity distribution. Reference 4 and a number of subsequent NACA reports, such as reference 7, contain velocity distributions for a large number of NACA base profiles. A convenient method of calculating the velocity distribution on base profiles is presented in reference 3. After the base-profile velocity distribution has been determined, the basic- and additional-lift distribution can be calculated by use of equations (24) and (31), respectively.

The total lift at a point for a given value of total-section-lift coefficient $c_l$ is

$$P_o = P_{b_o} + \left(c_l - c_{l_b}\right)\frac{P_{a_o}}{c_{l_a}} \quad \text{(equation (57) of reference 3)} \qquad (33)$$

where

$$c_{l_b} = \int_0^1 P_b \, d(x/c)$$

Equation (24), for the basic lift, can be solved numerically by either the method given in the appendix of reference 3 or by the method of reference 8. If, as is generally the case, the analytical relation between $\frac{V_f}{V_o}$ and $\theta$ is not known, equation (31), for the additional lift, can be solved graphically.

The inverse problem of calculating an airfoil section which will have a specified lift distribution can be solved in the manner outlined in reference 3. Equation (25), given herein, is used in determining the camber-line slope and can be solved by either the numerical method given in the appendix of reference 3 or by the method of reference 8.

## EVALUATION OF METHOD

An evaluation of the additional-, basic-, and total-lift distributions calculated by the method of this report is provided by comparing them with lift distributions obtained by the method of reference 3 and by experiment. Comparisons of additional-lift, basic-lift, and total-lift distributions are shown in figures 1, 2, and 3, respectively. The experimental data in figure 1(b) were obtained from reference 9 and the experimental data in the remainder of the figures were obtained from unpublished data from the Ames 7- by 10-foot wind tunnel. For the comparisons of basic-lift distributions (fig. 2), experimental data were not available for the ideal angle of attack as given by either the method of this report or by reference 3. Therefore, the available experimental data were interpolated to the ideal angles of attack as given by equation (21) and by reference 3.

From the comparisons shown in the figures, it is evident that the lift distributions calculated by equations (24) and (31) of this report are in better agreement with the experimental results shown than are the distributions calculated by the method of reference 3; it will be noted in particular that the agreement is better in the region near the airfoil leading edge. Further, although the method of this report is based on the assumption of small camber, as in the case of thin airfoil theory, the calculated basic-lift distributions for the sections with relatively large camber (fig. 2) are in good agreement with experiment.

## CONCLUDING REMARKS

Computation of lift distributions by the method of this report gives results which are in closer agreement with experimental distributions for the data examined than those obtained by use of the airfoil theory given in reference 3. The method of this report, in effect, modifies the method of reference 3 by adjusting the vorticity, given by thin-airfoil theory, for the local velocity induced by the airfoil-thickness distribution.

Ames Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Moffett Field, Calif., Dec. 12, 1950.

# APPENDIX

## NOTATION

| | |
|---|---|
| $A_o'$, $A_n$ | coefficients of the trigonometric series for distribution of vorticity |
| $c$ | section chord, feet |
| $c_l$ | section lift coefficient $\left(\frac{L}{qc}\right)$ |
| $L$ | total lift on the section, pounds |
| $P$ | local lift coefficient at any point $x$ |
| $q$ | free-stream dynamic pressure, pounds per square foot |
| $v$ | induced velocity at any point $x$, feet per second |
| $V_o$ | free-stream velocity, feet per second |
| $V_f$ | local velocity at the surface of the base profile, feet per second |
| $x$ | distance along chord, feet |
| $y_c$ | mean line ordinate, feet |
| $\alpha$ | angle of attack of the chord line, degrees |
| $\alpha_i$ | ideal angle of attack of the chord line, degrees |
| $\frac{d\Gamma}{dx}$ | rate of change of circulation, feet per second |
| $\theta$ | trigonometric coordinate, $\cos^{-1}\left[1-2\left(\frac{x}{c}\right)\right]$ |
| $\rho$ | density of free-stream air, slugs per cubic foot |

### Prescript

| | |
|---|---|
| $o$ | indicates application to airfoil of zero thickness |

## Subscripts

a indicates application to additional loading

b indicates application to basic loading

o indicates application to specific point along chord

REFERENCES

1. Theodorsen, Theodore, and Garrick, I. E.: General Potential Theory of Arbitrary Wing Sections. NACA Rep. 452, 1933.

2. Theodorsen, Theodore: Theory of Wing Sections of Arbitrary Shape. NACA Rep. 411, 1931.

3. Allen, H. Julian: General Theory of Airfoil Sections Having Arbitrary Shape or Pressure Distribution. NACA Rep. 833, 1945.

4. Abbott, Ira H., von Doenhoff, Albert E., and Stivers, Louis S., Jr.: Summary of Airfoil Data. NACA Rep. 824, 1945.

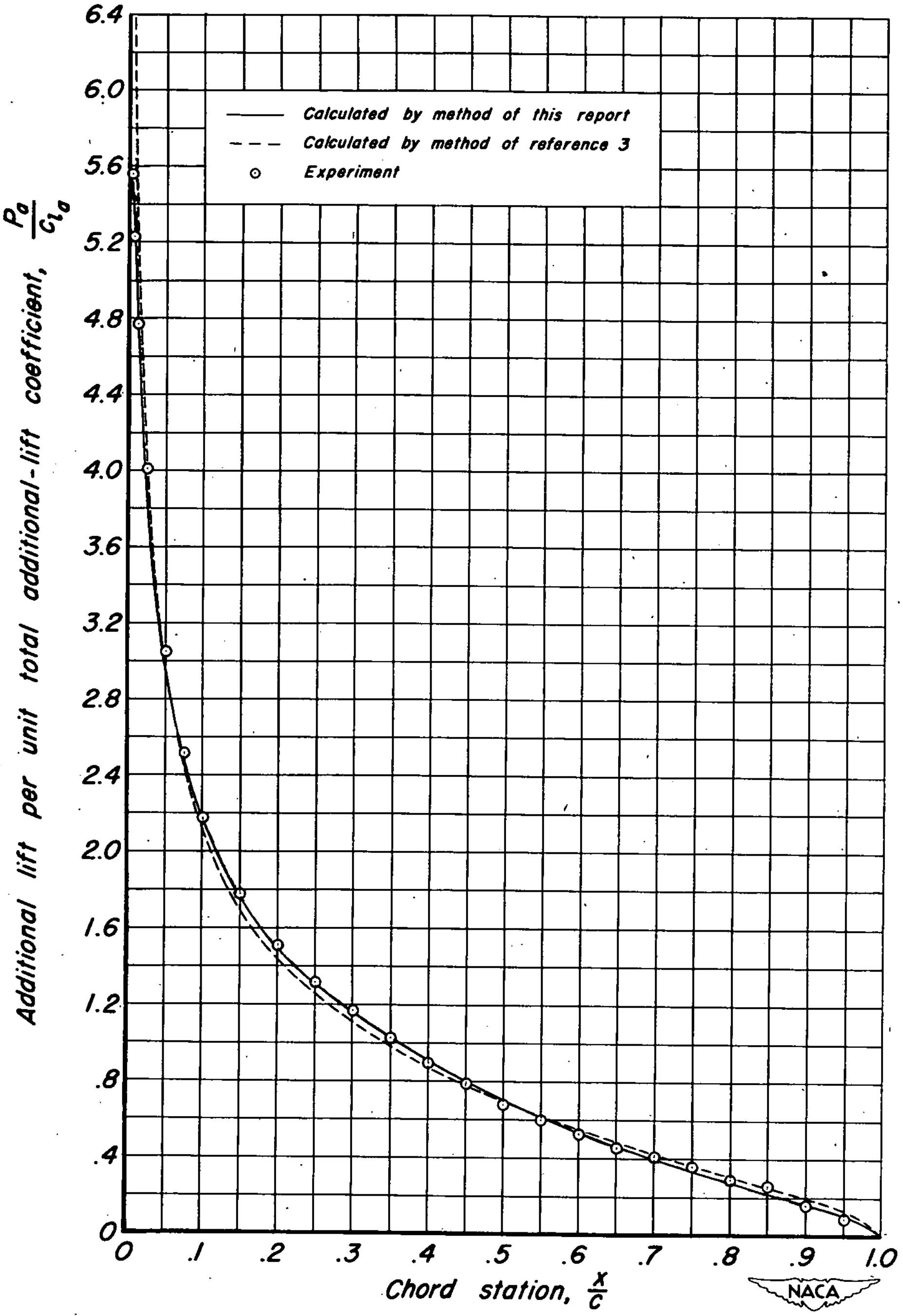
5. Mutterperl, William: The Conformal Transformation of an Airfoil Into a Straight Line and Its Application to the Inverse Problem of Airfoil Theory. NACA ARR L4K22a, 1944.

6. Jones, Robert T., and Cohen, Doris: A Graphical Method of Determining Pressure Distribution in Two-Dimensional Flow. NACA Rep. 722, 1941.

7. Loftin, Laurence K., Jr.: Theoretical and Experimental Data for a Number of NACA 6A-Series Airfoil Sections. NACA Rep. 903, 1948.

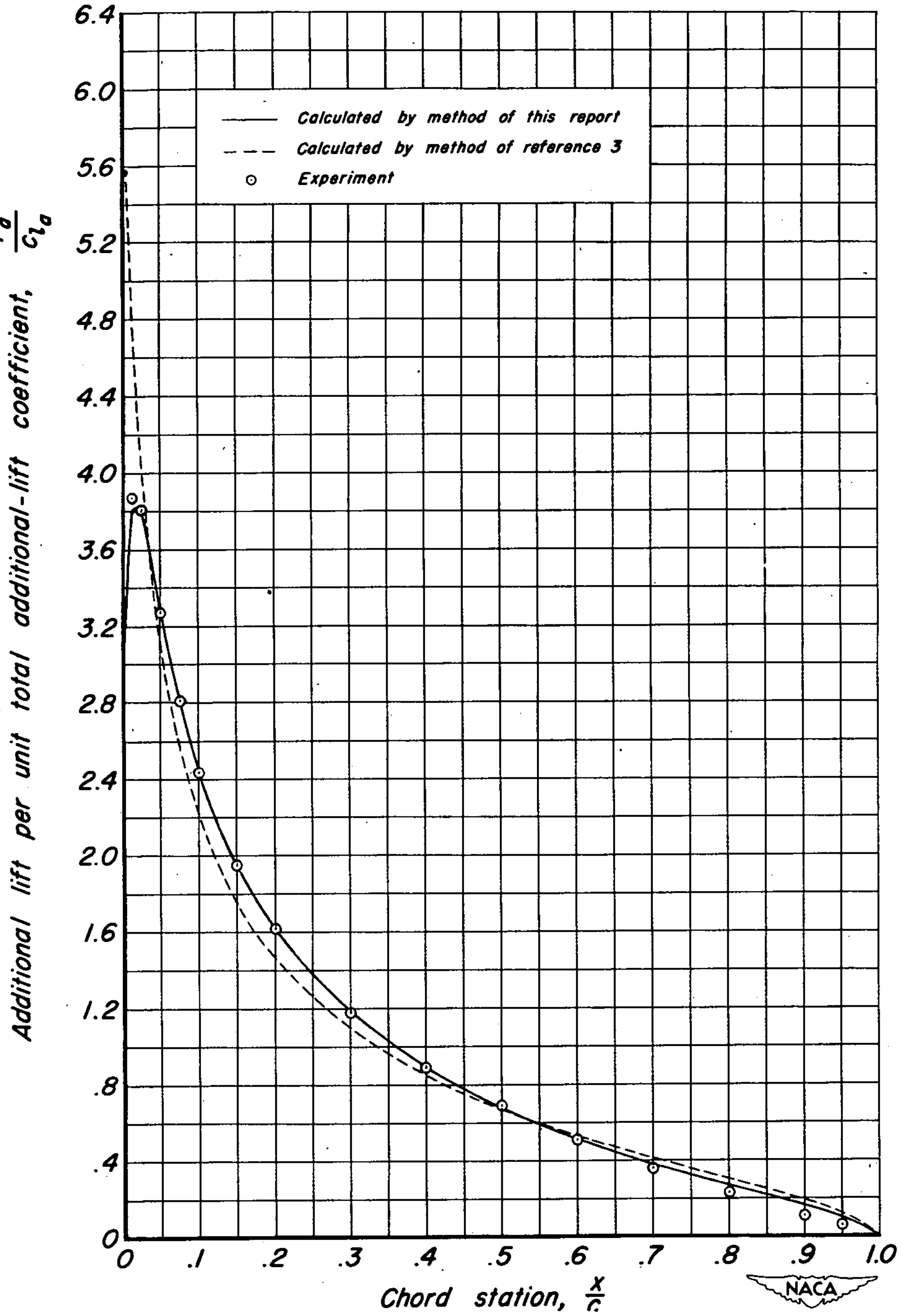
8. Naiman, Irven: Numerical Evaluation of the $\epsilon$-Integral Occurring in the Theodorsen Arbitrary Airfoil Potential Theory. NACA ARR L4D27a, 1944.

9. Jacobs, Eastman N., and Rhode, R. V.: Airfoil Section Characteristics as Applied to the Prediction of Air Forces and Their Distribution on Wings. NACA Rep. 631, 1938.



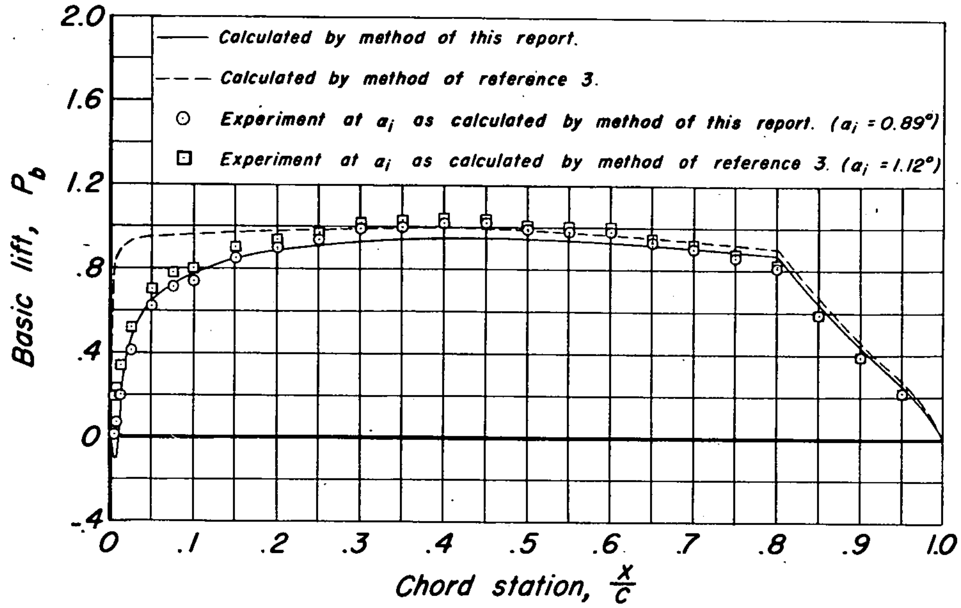

(a) NACA 63,012

Figure 1.- Comparison of the calculated and experimental additional-lift distributions.

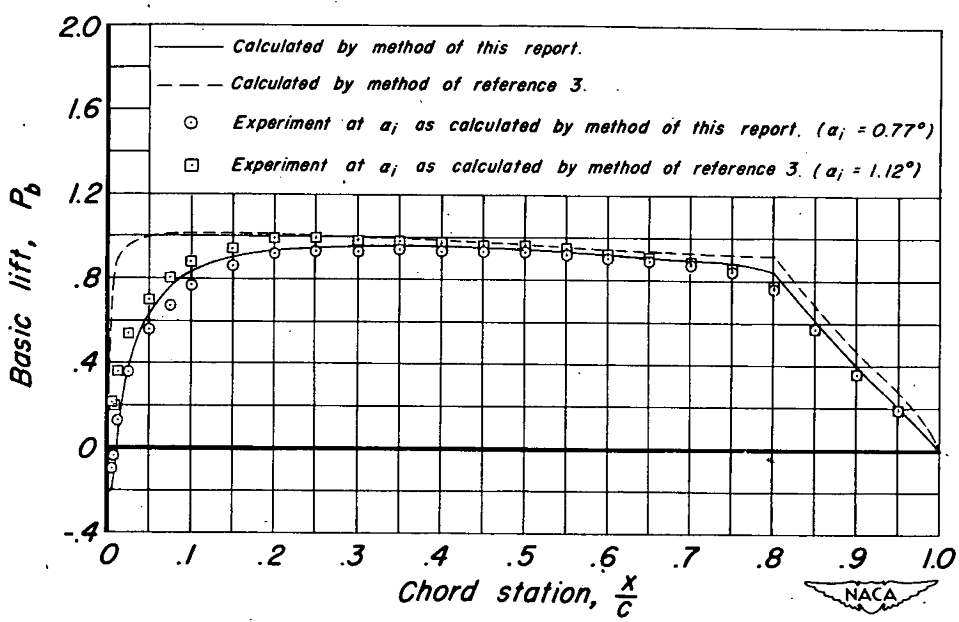



(b) NACA 0018

Figure 1. - Concluded.

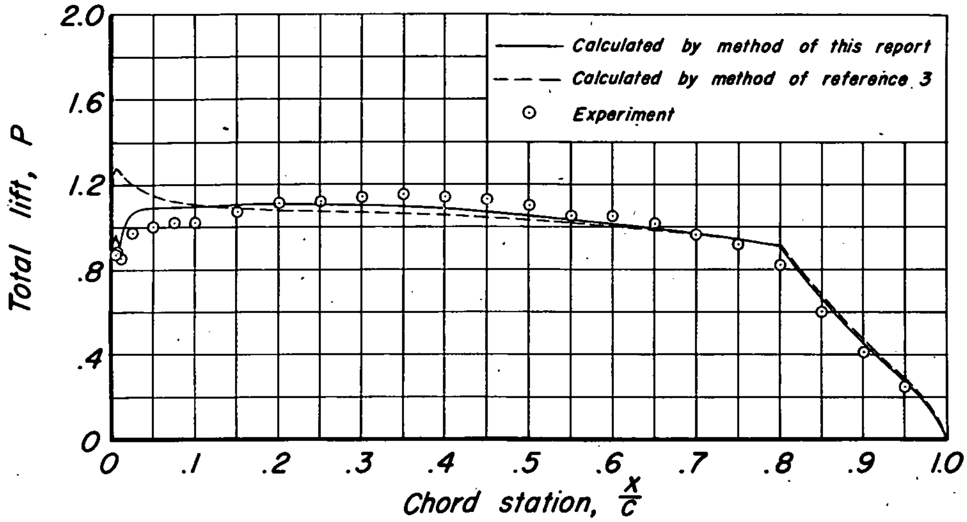



(a) NACA 64A810, a = 0.8 mod.

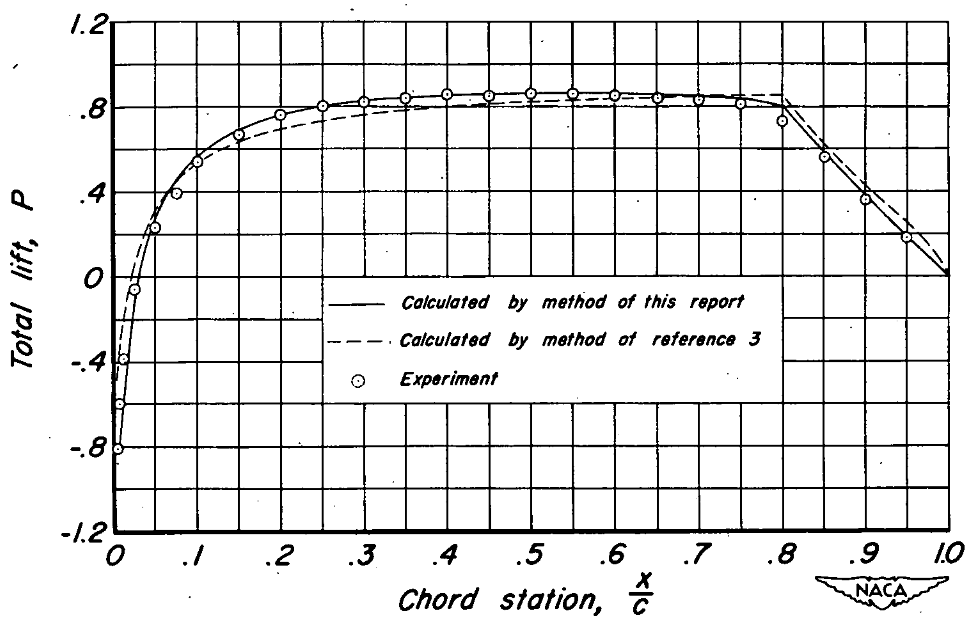



(b) NACA 0010-63, $c_{l_i}$ = 0.8, a = 0.8 mod.

Figure 2.- Comparison of calculated and experimental basic-lift distributions.

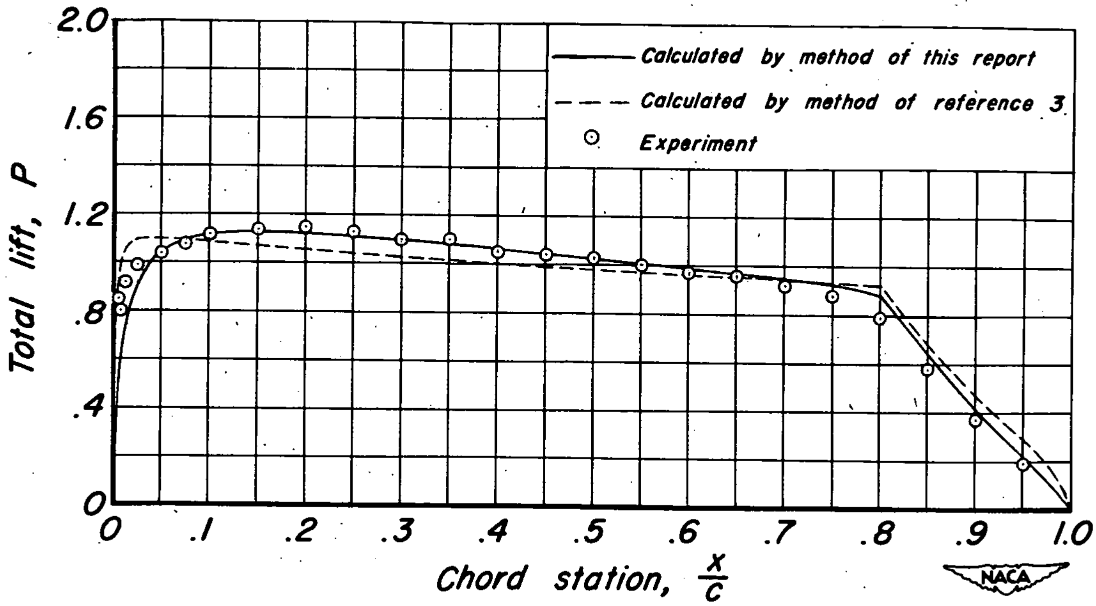



(a) NACA 64A810, $a = 0.8$ mod.; $c_l = 0.93$




(b) NACA 0010-63, $c_{l_i} = 0.8$, $a = 0.8$ mod.; $c_l = 0.65$

Figure 3.- Comparison of calculated and experimental total-lift distributions.




(c) NACA 0010-63, $c_{l_i} = 0.8$, $a = 0.8$ mod.; $c_l = 0.90$

Figure 3.- Concluded.

NACA-Langley - 2-20-51 - 950